
The Vasicek adjustment to beta estimates in the Capital Asset Pricing Model

17 June 2013

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1. Preparation of this report

This report was prepared by Professor Stephen Gray, Dr Jason Hall, Professor Robert Brooks and Dr Neil Diamond. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond acknowledge that they have read, understood and complied with the Federal Court of Australia's Practice Note CM 7, Expert Witnesses in Proceedings in the Federal Court of Australia. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond provide advice on cost of capital issues for a number of entities but have no current or future potential conflicts.

2. Executive summary

Beta estimates for the Capital Asset Pricing Model (CAPM) can be obtained by applying ordinary least squares (OLS) regression to the returns of firms in the same industry as the firm of interest and returns on a diversified market portfolio. However, these OLS estimates are known to be subject to a high degree of estimation error (Gray, Hall, Klease and McCrystal, 2009). Consequently, we consider whether an easily-implemented econometric technique, the Vasicek (1973) adjustment, can mitigate estimation error and thereby increase the reliability of beta (and ultimately cost of capital) estimates.

The Vasicek adjustment shifts the OLS beta estimate towards a prior expectation and the magnitude of that shift is greater when the standard error of the OLS estimate is higher. That is, where the OLS beta estimate is more precise it is given more weight, and when it is less precise it is given less weight. In our case the prior expectation of beta is one because, prior to conducting the regression analysis, we have no knowledge of whether the systematic risk of any given firm is above or below average, according to the rationale set out below.

Suppose we know nothing at all about the firm of interest, nothing about capital structure, industry, size, or competition.¹ Our best estimate would be that this firm has a beta of one, because we have no information whatsoever to determine whether it is a firm of above- or below-average systematic risk. Now suppose we are told the firm is in a particular industry and that it is financed with a specified amount of leverage. All else equal, leverage increases systematic risk, and different industries could have different exposure to economic conditions (i.e., different asset betas). What we cannot possibly know in advance is whether the combined impact of leverage and industry contributes to the firm having above- or below-average systematic risk. The firm could be in a low risk industry but have high leverage, or could be in a high risk industry but with low leverage. The reason we cannot know in advance whether the firm is likely to be relatively high or low risk is because we have not yet estimated beta for any similar firms. If we already knew the true beta for the average firm in the same industry with the same leverage, we would not need to perform the estimation at all.²

This is why estimates provided by the major commercial data vendors such as Datastream and Bloomberg adjust beta estimates towards a prior expectation of one. The estimates provided by Datastream incorporate a very similar adjustment to the Vasicek adjustment, with a prior expectation of one and the magnitude of adjustment contingent upon the standard error of the OLS beta estimate. Estimates provided by Bloomberg place a one-third weight on a prior expectation of one and a two-thirds weight on the OLS beta estimate.

¹ That is, suppose you are given the task of providing your best estimate of the beta of a particular firm, without being told which firm it is.

² That is, the task at hand is to select an appropriate beta value for the benchmark firm. The distribution of (re-levered) beta estimates for comparable firms can be used for that purpose. The goal is to construct the distribution of (re-levered) beta estimates for comparable firms. If we already knew that distribution somehow from our “prior” knowledge, there would be no need for any estimation at all.

We assess whether OLS and Vasicek-adjusted OLS estimates are able to predict stock returns when incorporated into the CAPM. Our objective is to determine whether we can analyse historical returns in order to make better predictions of future returns relative to just assuming that all stocks will earn the market return.

Each month we compile beta estimates using returns information from all prior months.³ We then form expected returns by incorporating these beta estimates into the CAPM, in which the risk free rate is the yield on 10 year government bonds and the market risk premium is the excess market return observed over the subsequent four weeks. In short, we ask, based upon the beta estimate and the market return over the month what return would we expect the stock to earn over that month? In performing this test we want to minimise the impact of random company- and industry-specific events on realised returns. So in each month we form portfolios in the following manner. First, we partition stocks into 10 industries according to the Industry Classification Benchmark (ICB) of FTSE. Second, within each industry group we partition stocks into equal-sized cohorts of high, medium and low beta stocks, formed on the basis of OLS beta estimates. We then aggregate the high beta stocks across the 10 industries, the medium beta stocks across the 10 industries and the low beta stocks across the 10 industries. This means that in each month we have three portfolios with the same industry composition, so mitigate the impact that company- and industry-specific events will have on the relative returns of each portfolio. We perform this analysis on two samples, a sample of 1,103 stocks for which at least 10 years of returns are available for beta estimation and a sample of 2,585 stocks for which at least 36 four-weekly returns are used in beta estimation (2.75 years of returns).

We find that expected returns formed from Vasicek-adjusted OLS estimates have more ability to explain the variation in realised stock returns than OLS estimates. For the sample in which at least 10 years of returns are available for analysis, in a regression of realised portfolio returns on expected returns the explanatory power is 57.78% as measured by the R-squared. In contrast, expected returns derived from OLS beta estimates have an R-squared of 56.59%. This is a small improvement in explanatory power, compared to assuming that all stocks have a beta of one. Under this naïve assumption the R-squared is 54.54%. Furthermore, when estimates are compiled using shorter time periods, OLS estimates perform worse than simply assuming all stocks have a beta estimate of one, but Vasicek-adjusted OLS estimates have some explanatory power.

These results do not imply that Vasicek-adjusted OLS estimates are highly reliable and should therefore be used in cost of capital estimation without consideration of other issues. The incremental explanatory power is low. But we can conclude that Vasicek-adjusted OLS estimates are more reliable measures of systematic risk than unadjusted OLS estimates.

³ Our returns are constructed over four weeks rather than every month, so each return is over the same time period, but we use the term monthly returns for convenience. There are 13.04 four-weekly returns per year, that is, $365.25 \text{ days} \div 28 \text{ days} = 13.04$ four-weekly periods.

3. Issue and evaluation approach

3.1. A statistical correction

The issue at hand is the usefulness of alternative techniques for estimating systematic risk, to be incorporated into the CAPM. We consider two estimation techniques which rely exclusively on the analysis of historical stock returns. The first estimation technique is OLS regression of excess stock returns on excess market returns. The second technique, the Vasicek adjustment, computes the beta estimate as a weighted average of the OLS estimate and a prior or “null” expectation.

The Vasicek-adjusted OLS estimates we compute are a weighted average of the OLS estimate and a beta estimate of one, the estimate for the average stock in the market. The idea behind these estimates is that we cannot observe the true beta for any stock, all we can observe is a beta estimate based upon a sample of returns. So if we observe a beta estimate that is well above or below that of the average stock, there is some chance that the stock really does have systematic risk which is very high or very low, and some chance that it has average risk and that we have observed the high or low beta estimate purely by chance – as a result of estimation error. In particular, a very low beta estimate is relatively more likely to have been affected by negative estimation error and a very high beta estimate is relatively more likely to have been affected by positive estimation error – even if estimation error is symmetric. This is explained in the context of the example that is set out in the Appendix.

Vasicek (1973) also demonstrates that a beta estimate that is below one is more likely to have been affected by negative estimation error. He also shows that the further a beta estimate is below one, the greater the likely impact of negative estimation error. Consequently, if the OLS beta estimate for a particular firm is below one, the best forecast of the true beta for that firm is something above the OLS estimate. Vasicek (1973) demonstrated that the adjustment to eliminate the effect of estimation error depends upon the standard error of the beta estimate. So the Vasicek-adjusted estimate places some weight on the OLS estimate, and some weight on a prior estimate formed prior to analysing the stock returns, and those weights depend on the standard error of the beta estimate.

3.2. The statistical “prior”

In the case at hand, the statistical prior estimate is one, which is (by definition) the true beta of the average firm. Consequently, our Vasicek-adjusted beta estimates place some weight on the OLS beta and some weight on the average or “null” beta estimate of one – where the relative weights depend upon the statistical precision of the OLS estimate.

In our setting, the goal is to estimate the beta of the average firm in a particular industry. In other settings, the goal is to estimate the beta of a specific individual firm. If the researcher already knows the beta of the average firm in a particular industry, the Vasicek approach can be applied by adjusting the OLS estimate for an individual firm towards the beta of the average firm in the industry. But this is not at all relevant in our setting – if we knew the beta of the average firm in the industry, the task

would already be complete and no further analysis would be required. The whole point is that we don't know the beta of the average firm in the industry – that is what we are seeking to estimate. The only quantitative a priori information that is available is that the beta of the average firm is one, by definition.⁴

There are at least two data providers who compile beta estimates that are consistent with the Vasicek approach (whereby raw OLS estimates are adjusted towards 1.0 based on the standard error of the OLS estimate). The default estimates provided by Datastream are compiled using a Vasicek adjustment, according to the technique described in Cunningham (1973). Bloomberg reports beta estimates after applying a one-third weight to a prior estimate of one, and a two-thirds weight to the OLS estimate.

3.3. Statistical rationale

Blume (1975) also documented mean reversion in OLS beta estimates, in that if we observe a low beta estimate from sample returns in one period, we are likely to observe a higher beta estimate next period, and that high beta estimates predict relatively lower beta estimates next period. Blume speculated that one reason for this mean-reversion could be that over time the systematic risk of the firm's portfolio of assets approaches the systematic risk of the average firm in the market. But Blume merely offered this as a potential explanation and did not test this conjecture. Vasicek demonstrated that, even if there were no change whatsoever to the firm's true systematic risk, we would observe mean-reversion in beta estimates.

That is, there are two rationales for computing an adjusted beta. First, the true beta may revert towards one over time as the firm takes on more projects with systematic risk which is about the same as the average firm; or the firm changes its financing structure such that firms with high operating leverage take on less financing leverage and vice versa. Under this rationale, having obtained a low beta estimate this period, we would predict a higher beta estimate next period on the basis that the firm will undertake actions that cause its true beta to revert towards one.

The second rationale is purely statistical, as set out in the example in the appendix. Under this rationale, having observed a low beta estimate this period, we apply a statistical adjustment to correct for statistical bias caused by estimation error. Note that under this rationale, the true beta does not revert towards one over time at all – the adjustment is performed as a statistical correction only. As set out in the appendix, the Vasicek correction is clearly a statistical adjustment to correct bias due to

⁴ In Vasicek's (1973) paper he provides an example of the case in which the prior expectation is not equal to one, and refers to the case of a utility in which the prior expectation is 0.8 with a standard error of 0.3. There are two points to make with reference to this example. First, Vasicek was providing the reader with an example merely to describe the case in which the prior expectation is something different from one and elected to illustrate this point with a utility example. He was not making a comment about the actual beta estimates for utilities. Second, Vasicek was also illustrating the general case in which a prior expectation can only be formed using information other than the data and technique used in beta estimation. This means that it is not appropriate to use the same data and technique to form the prior expectation and conduct the regression analysis.

estimation error, as it is based on Bayes' Rule. Such a correction clearly has nothing to do with true betas mean reverting over time.

The Australian Energy Regulator (AER, 2009) has previously not adopted the Vasicek adjustment for the small sample of Australian-listed stocks it typically considers. The AER's primary reason for the rejection of any adjustment towards one is that the benchmark firm does not diversify or change its capital structure over time, so would not be expected to have a true beta that reverts towards one. We agree that the mean reversion rationale is unlikely to apply to the benchmark firm. This is our motivation for using the Vasicek correction; it mitigates the statistical bias caused by estimation error – based on Bayes' Rule, as set out in the example in the appendix.

The AER has also noted that the actual sample firms generally had beta estimates below one and so adopting a prior estimate of one was likely to lead to an upwards bias in the results (p.297).⁵ It contends that because it has used a sample of stocks in the same industry it has accounted for the potential estimation error in OLS beta estimates.

The problem is that the AER has not addressed the real possibility that a very low OLS estimate is more likely to understate risk than to overstate risk, and a very high OLS estimate is more likely to overstate risk than understate risk. The use of a sample of firms in the same industry does not account for this. If an event occurs which affects the industry in general, this can lead to an increase or decrease in the beta estimates across the industry. For example, if there is takeover activity in a particular industry when the market is performing well, the beta estimates for those firms will increase because industry returns are high when the market returns are high; if there is a downturn in commodity prices when the market is performing well the beta estimates for resources companies will decrease because industry returns are low when the market is performing well.

3.4. Performance evaluation

To evaluate the performance of OLS beta estimates and Vasicek-adjusted OLS beta estimates we tested the extent to which expected returns formed on the basis of a particular beta estimate are able to predict future stock returns – relative to expected returns that are formed on the basis that all stocks have a beta of one (the beta of the average firm). In other words, an estimate will be relatively more reliable if it allows us to predict stock returns better than simply assuming all stocks have a beta of one. Testing estimates against this criteria is generally challenging, because of the volatility of stock returns. However, we have compiled a sample size that is sufficiently large to enable us to detect improvements in the reliability of the estimates relative to a statistical prior assumption that beta is equal to one.

We measure this improvement by comparing the *R*-squared from the regression of expected returns on actual stock returns – for portfolios formed from high, medium and low beta stocks.

⁵ The first comment was made with respect to the Blume adjustment (in which a constant weight is applied to a prior expectation instead of a weight which varies according to the standard error of the beta estimate) but this same rationale was used to reject the use of the Vasicek adjustment.

Expected returns are formed by observing actual market returns and actual risk-free rates and asking, “Given the beta estimate and the actual return earned by the market, what is the return we would expect to earn on the portfolio?” This expected return appears on the right hand side of the regression and the realised return for the portfolio appears on the left hand side. The reason we form portfolios for this analysis is to eliminate the noise from individual stock returns as much as possible.

The R -squared from the regression measures how much the variation in realised returns can be explained by expected returns formed on the basis of a particular estimate of beta. An R -squared of 100 per cent would imply that the CAPM (based on the particular estimate of beta from historic return observations) could predict future returns with certainty. An R -squared of 50 per cent means that the CAPM (based on the particular estimate of beta) can explain half of the variation in actual stock returns, with the remaining variation in stock returns being explained by risk factors not encapsulated by the CAPM.

We find that Vasicek-adjusted OLS estimates are better able to explain realised returns than are raw OLS estimates. While there is only a small improvement in explanatory power over the assumption that all stocks have beta equal to one, the evidence shows that Vasicek-adjusted OLS estimates are relatively more reliable.

3.5. Detailed methodology

We first compute OLS beta estimates according to the following equation. Every four weeks we compile beta estimates using returns information from all prior periods.⁶ In OLS regression, the intercept (α) and coefficient on excess market returns (β) is determined in order to minimize the sum of squared errors.

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t}$$

where:

$r_{i,t}$, $r_{m,t}$ and $r_{f,t}$ represent the return on stock i , the return on the equity market and the risk free rate, respectively in period t ; and

$\varepsilon_{i,t}$ represents the error term for stock i during period t .

Vasicek-adjusted OLS estimates place some weight on a statistical “prior,” and some weight on the OLS estimate, with weights determined by the standard error of the OLS estimates. As explained above, we adopt a beta of one as our statistical prior. This beta value is equal to the systematic risk of the average firm (by definition). We compute Vasicek-adjusted OLS estimates for stock i (β_i^{VAS}) according to the following equation, in which the standard error of our statistical prior $E[std\ err]$ is estimated as the standard deviation of OLS beta estimates in that month:⁷

⁶ Our returns are constructed over four weeks rather than every month, so each return is computed over the same time period.

⁷ This is likely to overstate the standard error of the prior estimate and therefore likely to understate the extent to which the OLS estimates should be shifted towards one on the basis of the Vasicek adjustment. The reason it is likely to overstate the standard error of the prior estimate is because the standard deviation of OLS estimates

$$\beta_i^{VAS} = \beta_i^{OLS} \times \frac{E[\text{std err}]^2}{E[\text{std err}]^2 + \text{Std err of OLS est}^2} + \left(1 - \frac{E[\text{std err}]^2}{E[\text{std err}]^2 + \text{Std err of OLS est}^2}\right)$$

Having compiled beta estimates under both techniques, we then evaluate whether expected returns formed with respect to these estimates are able to explain the variation in realised returns. We then form expected returns by incorporating these beta estimates into the CAPM, in which the risk free rate is the yield on 10-year government bonds and the market risk premium is the excess market return observed over the subsequent four weeks. In short, we ask, based upon the beta estimate and the market return over the month what return would we expect the stock to earn over that month. For example, if the beta estimate was 0.8, government bond yields were 0.5% and the market return was 2.0% we would expect the stock to earn returns of 1.7%, computed as $r_f + \beta \times (r_m - r_f) = 0.005 + 0.8 \times (0.020 - 0.005) = 0.005 + 0.8 \times 0.015 = 0.005 + 0.012 = 1.7\%$.

In performing this test we seek to minimise the impact of random company- and industry-specific events on realised returns by forming portfolios in the following manner. First, we partition stocks into 10 industries according to the Industry Classification Benchmark (ICB) of FTSE. Second, within each industry group we partition stocks into equal-sized cohorts of high, medium and low beta stocks, formed on the basis of OLS beta estimates. We then aggregate the high beta stocks across the 10 industries, the medium beta stocks across the 10 industries and the low beta stocks across the 10 industries. This means that in each month we have three portfolios with the same industry composition, so mitigate the impact that company- and industry-specific events will have on the relative returns of each portfolio. We perform this analysis on two samples. We consider a sample for which at least 131 four-weekly returns are used in beta estimation (10 years of returns) and a sample for which at least 36 four-weekly returns are used in beta estimation (2.75 years of returns).

reflects both the dispersion of true betas and the estimation error inherent in the OLS estimates. So the standard deviation of beta estimates will be wider than the standard deviation of true betas.

4. Data

We compiled beta estimates for 2,585 Australian-listed stocks using returns computed from 2 January 1976 to 4 May 2012. The returns interval is four weeks, computed using Friday closing prices, so there are 474 four-week periods. The market index is the All Ordinaries Accumulation Index from 1 May 1992 and the Datastream Australia Total Market Index prior to this date. The estimate of the risk free rate is the yield to maturity on 10-year Australian government bonds as reported by the Reserve Bank of Australia, converted to a 28 day yield. We excluded firms with less than 36 four-weekly returns observations and individual observations with a four-weekly return of more than 200%.⁸

The beta estimates at each date comprise all available returns information prior to that date. In our test of the relationship between expected returns and realised returns, we perform the analysis using two samples. One sample requires at least 131 returns observations to be used in beta estimation, and the other requires at least 36 returns observations to be used. There are two reasons we evaluate the results over two time periods. First, we want to document whether the results differ if shorter or longer time periods are used in beta estimation. Second, in practice, beta estimates are often compiled over different time periods, either because there is limited data actually available or because the analyst determines that a particular time period will provide more relevant information.

The selection of a minimum period of 10 years for the first sample aligns with the period from January 2002 to the present day. In prior analysis, the AER has compiled beta estimates which exclude the technology bubble which the AER determined ended at the end of 2001. The selection period of a minimum of 36 returns for the second sample, which equates to 2.75 years, was chosen because as a rule of thumb practitioners often require at least 36 returns observations before including a firm in their sample. Some practitioners require 24 observations, others 48 or 60, but this appears to be a reasonable minimum period for our analysis.

To ensure that the tests are performed over the same period of time, the first beta estimates are compiled from 17 January 1986, which allows at least 131 returns observations, to 6 April 2012. The first four-weekly period of realised returns ends on 14 February 1986 and the last period ends on 4 May 2012.

The final sample comprises 247,652 beta estimates from 2,585 firms. On average, each beta estimate is compiled using 125 returns periods in estimation, equivalent to 9.6 years of returns. The sample which requires at least 10 years of returns information to be used in beta estimates has 93,101 beta estimates from 1,103 firms. On average each beta estimate is compiled using 206 returns periods in estimation, equivalent to 15.8 years of returns.

⁸ Application of this filter results in the exclusion of less than 0.5% of observations. Some of these observations will represent returns of stocks which happen to be volatile. Other cases will represent data errors and the assumptions used by Datastream in accounting for changes in capitalisation. The most extreme stock return prior to the application of this filter was 15,133% for Equatorial Resources on 3 May 2002.

Table 1. Beta estimates

	Mean	Std Dev	Percentiles (%)				
			5 th	25 th	50 th	75 th	95 th
Panel A: At least 131 periods used in beta estimation (N = 93,101; 1,103 firms)							
OLS estimate	0.89	0.53	0.16	0.49	0.82	1.24	1.87
Vasicek estimate	0.87	0.43	0.22	0.55	0.84	1.19	1.59
Panel B: At least 36 periods used in beta estimation (N = 247,652; 2,585 firms)							
OLS estimate	0.96	0.77	-0.01	0.45	0.85	1.37	2.35
Vasicek estimate	0.91	0.46	0.21	0.58	0.90	1.23	1.71

The sample comprises four-weekly return observations from 2,585 stocks listed on the Australian Stock Exchange from 2 January 1976 to 4 May 2012 for which at least 36 returns observations are available. From 1 May 1992 the market index is the All Ordinaries Index. Prior to this date the market index is the Datastream Australia Total Market Index. The risk free rate is the yield to maturity on 10 year Australian government bonds, converted to a four-weekly rate.

In Table 1 we summarise the distribution of beta estimates. For the sample in which at least 10 years of returns information is used, the mean OLS beta estimate is 0.89⁹ with a standard deviation of 0.53. Despite the long estimation period there is substantial dispersion of OLS beta estimates. Half of the OLS beta estimates lie outside the range of 0.49 to 1.24, and 10 per cent of estimates are either above 1.87 or below 0.16.

Vasicek-adjusted estimates have a mean value of 0.87 and a standard deviation of 0.43 when estimated using 10 years of returns. When 36 months of returns are used in the analysis, we observe a mean estimate of 0.91 and a standard deviation of 0.46. The reason the standard deviations are approximately the same in both sets of data is because the impact of the Vasicek adjustment is greater when the estimates have a high standard error. When fewer returns are used in estimation the standard error of the estimates is larger, so the Vasicek adjustment will have more impact. Across the returns distribution, the percentiles are approximately the same regardless of the estimation period. For example, the 5th percentile of the Vasicek-adjusted estimates is 0.22 when 10 years of returns are used, and is 0.21 when 36 months of returns are used. At the 95th percentile the corresponding figures are 1.59 and 1.71.

The key point for practical purposes is that the Vasicek adjustment is relatively small when the OLS estimate has a low standard error, which is a feature of estimates constructed using a long time series. The idea behind the Vasicek adjustment is that for estimates at the extremes of the distribution, there is a high probability that these estimates resulted from noise in the data, rather than representing the true systematic risk of the stock. Take the stocks at the 5th percentile of the distribution, estimated using 36 months of returns. It is unlikely that 5% of stocks truly have less systematic risk than government bonds. It is more likely that these 5% of stocks have beta *estimates* that are statistically unreliable and which have been affected by negative estimation error.

⁹ This is the equal-weighted average of beta estimates across firms, so would not be expected to equal one.

Table 2. Industry beta estimates

Industry	At least 131 periods used in estimation			At least 36 periods used in estimation		
	N	OLS	Vasicek	N	OLS	Vasicek
Oil & Gas	7,207	1.19	1.14	18,216	1.19	1.10
Basic Materials	29,140	1.15	1.09	78,821	1.21	1.10
Industrials	14,831	0.70	0.70	35,053	0.78	0.78
Consumer goods	8,485	0.54	0.56	18,127	0.59	0.63
Health care	4,412	0.95	0.92	14,349	1.06	1.01
Consumer services	7,288	0.72	0.74	20,514	0.78	0.78
Telecommunications	685	1.46	1.30	2,896	1.49	1.24
Utilities	1,549	0.82	0.83	3,369	0.85	0.85
Financials	16,188	0.66	0.68	45,163	0.69	0.72
Technology	3,316	1.10	1.06	11,144	1.25	1.10
Full sample	93,101	0.89	0.87	247,652	0.96	0.91

The table comprises mean beta estimates across 10 industry super-sectors formed according to the International Classification Benchmark of FTSE.

Recall that in our empirical analysis we construct portfolios of high, medium and low beta estimates for portfolios with the same industry composition. In Table 2, we present the mean beta estimates across the ten industry groups. The impact of the Vasicek adjustment is most prevalent when a minimum of just 36 periods is used in estimation. On an industry basis the average adjustment is 0.07, with the biggest adjustment of 0.25 occurring in Telecommunications. When at least 10 years of returns information is used in estimation the largest adjustment to an industry average is 0.16 and the average adjustment is 0.04.

5. Relationship between expected returns and realised returns

In this section we document the relationship between expected returns and realised returns, for portfolios in which expected returns are formed on the basis of their beta estimates, government bond yields and market returns. Recall that the portfolios of high, medium and low beta stocks are formed with approximately the same industry composition, and the only difference across the three portfolios is the beta estimate used in forming expected returns.¹⁰ As a benchmark, we compare explanatory power with the case in which expected returns are simply equal to the market return. In other words, we examine whether there is incremental explanatory power over the assumption that all stocks have a beta estimate of one. In Table 3 we summarise the distribution of sample returns and in Table 4 we document the results of a regression of portfolio returns on expected returns.

For the sample in which at least 10 years of returns are available for beta estimation the mean stock return is 1.22%, the mean market return is 0.73% and the mean risk free rate is 0.47%. On an annualised basis the mean returns are 17.1% for stocks, 10.0% for the market and 6.3% for the risk free rate.¹¹ These descriptive statistics are compiled with respect to the individual return observations so there are more observations in recent years when the market return was relatively low. Over the entire sample period, including the period used in beta estimation, the average four-weekly market return was 1.06% (annualised 14.7%) and the average risk-free rate was 0.67% (annualised 9.2%). The standard deviation of stock returns is 20.06%, compared to 4.40% for the market. On an annualised basis this represents volatility of 72.5% for stocks and 15.9% for the market. The distribution of returns for the larger sample, in which at least 36 returns are required for beta computation, is not substantially different from the larger sample.

Turning to the regression results reported in Table 4, consider first the performance of expected returns formed on the basis of OLS estimates. When we consider only observations in which at least 10 years of returns are available for beta estimation, expected returns are able to explain 56.59% of the variation in realised portfolio returns. This is an improvement over the explanatory power of the market itself, which is able to explain 54.54% of the variation in returns. The *R*-squared increases to 57.78% in the case of Vasicek-adjusted OLS estimates. So for the sample in which a long time series of returns is available for analysis, Vasicek-adjusted estimates offer the greatest improvement in explanatory power over the market return itself.

¹⁰ We cluster standard errors by portfolio because the same firm will often appear in the same portfolio over time, so the observations are not independent.

¹¹ For example, the annualised return for stocks is computed as $(1.0122)^{(365.25/28)} - 1 = 0.171$.

Table 3. Distribution of returns (%)

	Mean	Std Dev	Percentiles (%)				
			5 th	25 th	50 th	75 th	95 th
Panel A: At least 131 periods used in beta estimation (N = 93,101)							
Stock return	1.22	20.06	-25.00	-7.98	0.00	7.15	33.31
Market return	0.73	4.40	-6.21	-1.57	1.21	3.38	6.51
Risk free rate	0.47	0.14	0.31	0.40	0.43	0.48	0.78
Excess stock return	0.74	20.06	-25.43	-8.43	-0.45	6.70	32.73
Excess market return	0.25	4.40	-6.62	-2.01	0.75	2.94	9.68
Panel B: At least 36 periods used in beta estimation (N = 247,652)							
Stock return	1.05	18.21	-23.18	-6.73	0.00	6.40	28.57
Market return	0.69	4.74	-6.36	-1.96	1.29	3.54	6.58
Risk free rate	0.48	0.17	0.31	0.40	0.43	0.47	0.97
Excess stock return	0.57	0.18	-23.62	-7.22	-0.42	5.91	28.12
Excess market return	0.21	4.73	-6.79	-2.59	0.76	3.08	6.09

In this table we summarise the distribution of stock returns, market returns and the risk free rate used in our test of whether expected returns are associated with market returns. Returns are computed over 343 four-week periods with the first period ending on 17 January 1986 and the last period ending on 4 May 2012. Summary statistics are computed using all firm-period observations. The sample considered in Panel A comprises 93,101 observations from 1,103 unique stocks in which at least 131 periods of returns (10 years equivalent) were available for beta estimation. The sample considered in Panel B comprises 247,652 observations from 2,585 unique stocks in which at least 36 periods of returns (2.75 years equivalent) were available for beta estimation.

For the larger sample in which at least 36 periods of returns are required for estimation, we observe no incremental ability of expected returns derived from OLS estimates to explain portfolio returns. If we impose the naïve assumption that all stocks have a beta estimate of one, we are able to explain 49.55% of the variation in portfolio returns. The *R*-squared associated with OLS estimation is 47.88%. In contrast, expected returns derived from Vasicek-adjusted OLS estimates are able to explain 51.88% of the variation in portfolio returns.

We examined whether our conclusions were sensitive to the inclusion or exclusion of a small number of periods when market returns were unusually high or low. The 343 four-week periods considered included one instance in which the market return was -34.4%, one instance in which the market return was -22.3% and a further nine instances in which the market return was either below -10% or above +10%. We repeated our analysis after excluding both the highest and lowest market returns one at a time for up to 10 excluded periods, and observed the incremental change in explanatory power. These results are presented in Table 5. In every instance we observe the same relative explanatory power across the estimation techniques. Expected returns derived from Vasicek-adjusted estimates have the highest explanatory power for both samples and for the larger sample, this is the only measure of expected returns which has more explanatory power than the market itself.

Table 4. Regression of portfolio returns on expected returns

Method	Intercept (%)	$p H_0 = 0$	Coefficient	$p H_0 = 1$	R-squ (%)
Panel A: At least 131 four-weekly returns (10 years)					
OLS	0.35	0.01	0.96	0.72	56.59
Vasicek	0.31	0.00	1.01	0.94	57.78
Market	0.38	0.07	0.84	0.37	54.54
Panel B: At least 36 four-weekly returns (2.75 years)					
OLS	0.57	0.04	0.88	0.44	47.88
Vasicek	0.47	0.03	1.00	0.99	51.88
Market	0.54	0.06	0.88	0.44	49.55

On each estimation date we disaggregated the sample into 10 ICB industry super-sectors and within each super-sector allocated stocks into three equal-sized cohorts according to high, medium and low OLS beta estimates. We then aggregated stocks across the ten sectors into the high, medium and low OLS beta cohorts, meaning that our portfolios have approximately equal industry composition. Within each portfolio we computed mean OLS and Vasicek-adjusted beta estimates, and then formed conditional expected returns according to the equation: Expected return = Risk free rate + Beta \times (Market return – risk free rate). The risk free rate is the yield to maturity on 10 year government bonds at the portfolio formation date and the market returns is the return observed in the four weeks subsequent to the portfolio formation date. We then regress mean realised returns against conditional expected returns. In Panel A we present results for the case where stocks are only included if at least 131 four-weekly returns are available for beta estimation, equivalent to 10 years of returns data. In Panel B we present results for the case where stocks are only included if at least 36 four-weekly returns are available for analysis. There are 1,029 returns observations which comprises 3 portfolios \times 343 four-weekly returns from 17 January 1986 to 4 May 2012. Standard errors are clustered by cohort (high, medium and low beta cohort).

Table 5. R-squared values as observations from large market movements are removed

Periods	343	341	339	337	335	333
Panel A: At least 131 four-weekly returns (10 years)						
OLS	56.59	53.73	50.63	50.04	48.42	48.41
Vasicek	57.78	54.99	51.68	51.06	49.33	49.38
Market	54.54	52.82	49.58	48.57	46.79	46.91
Panel B: At least 36 four-weekly returns (2.75 years)						
OLS	47.88	45.11	42.08	41.51	39.87	40.66
Vasicek	51.88	49.36	46.07	45.47	43.73	44.69
Market	49.55	48.16	45.03	44.03	42.28	43.56

In this table we present R-squared values for cases in which we incrementally remove observations with the highest and lowest market returns over four-weekly periods. In each instance we remove two periods which corresponds to six observations, as each period is associated with a portfolio of high, medium and low beta estimates.

6. Conclusion

Four decades ago, researchers documented an important property of beta estimates from OLS regression. Low beta estimates are likely to understate systematic risk and high beta estimates are likely to overstate systematic risk. This was established both in statistical theory and empirical practice. Estimates generated by Datastream and Bloomberg account for this tendency, by placing some weight on the market average beta of one and some weight on the OLS beta estimate. The Datastream adjustment is contingent upon the standard error of the beta estimate, while the Bloomberg estimate places constant weight on a prior estimate of one.

The estimate we examine is the Vasicek-adjusted OLS estimate, which like the Datastream estimate adjusts the OLS estimate towards one, with the magnitude of the adjustment contingent upon the standard error. In the manner we have computed Vasicek-adjusted OLS estimates, it is easy to implement and requires no subjective judgment. So for OLS estimates with a high standard error, it offers a convenient technique for mitigating well-known statistical estimation error.

The primary argument against the use of the Vasicek adjustment is simply, “Why is the prior expectation equal to one? Why isn’t the prior expectation something else, like the industry average?” This argument can only be made if we know what the “something else” is, like the industry average. But prior to measuring systematic risk we do not know whether the prior expectation of risk is higher or lower than the average firm. If we have not yet measured systematic risk, how could we possibly form a view that the combined impact of operating and financial risk leads to the firm’s beta being above or below one?

Given there is support in theory, evidence and in practice for the Vasicek adjustment, the question is whether it is useful. Does it generate beta estimates that are more reliable than OLS estimates? The evidence suggests that it does. Vasicek-adjusted OLS estimates, when incorporated into the CAPM to form expected returns, are able to predict stock returns to a limited degree. The Vasicek-adjusted OLS estimates have more predictive ability than OLS estimates, but the improvement in explanatory power is small. The implication is that cost of capital estimates formed using the Vasicek adjustment are likely to be more reliable than estimates formed using unadjusted OLS estimates. There appears to be no sound reason for not using this adjustment in estimating systematic risk.

7. References

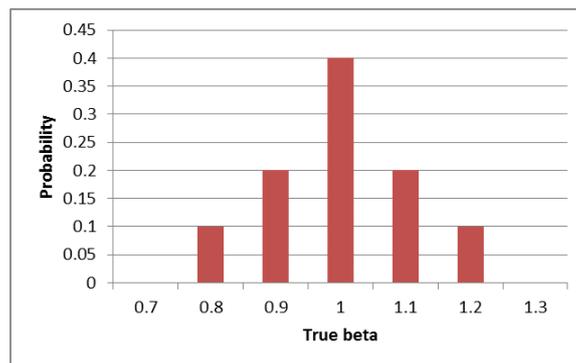
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8. Appendix

To illustrate the statistical correction for estimation error via the application of Bayes' Rule, consider the following example.

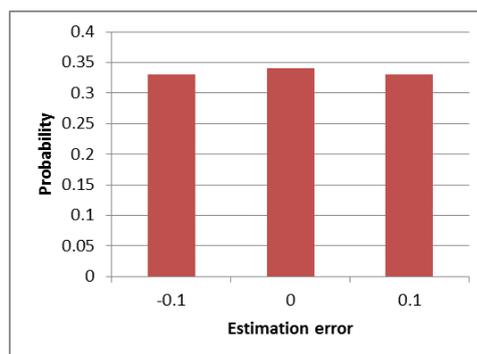
Suppose the distribution of **true** betas is as follows:

True beta	Probability
0.8	0.1
0.9	0.2
1.0	0.4
1.1	0.2
1.2	0.1



That is, true betas are distributed symmetrically around one. Also, suppose estimation error is distributed as follows:

Estimation error	Probability
-0.1	1/3
0.0	1/3
0.1	1/3



That is, 40% of the stocks in the market have a true beta of 1.0. If you select one of those stocks and find an **estimate** of its beta, there is a one third chance that the estimate will turn out to be 1.0, a one third chance that the estimate will turn out to be 0.9 and a one third chance that the estimate will turn out to be 1.1.

Now suppose that we want to find the best estimate of the true beta for a particular firm, given that its beta *estimate* is 0.9. In this case, we know that there are three possible ways to obtain a beta estimate of 0.9:

1. The true beta is 0.8 and estimation error is 0.1;
2. The true beta is 0.9 and estimation error is 0; or
3. The true beta is 1.0 and estimation error is -0.1.

Bayes' Rule states that:

$$\Pr[\beta = 0.8 | \hat{\beta} = 0.9] = \frac{\Pr[\hat{\beta} = 0.9 | \beta = 0.8] \Pr[\beta = 0.8]}{\Pr[\hat{\beta} = 0.9]} = \frac{0.33 \times 0.1}{0.233} = 0.1429$$

$$\Pr[\beta = 0.9 | \hat{\beta} = 0.9] = \frac{\Pr[\hat{\beta} = 0.9 | \beta = 0.9] \Pr[\beta = 0.9]}{\Pr[\hat{\beta} = 0.9]} = \frac{0.33 \times 0.2}{0.233} = 0.2857$$

$$\Pr[\beta = 1.0 | \hat{\beta} = 0.9] = \frac{\Pr[\hat{\beta} = 0.9 | \beta = 1.0] \Pr[\beta = 1.0]}{\Pr[\hat{\beta} = 0.9]} = \frac{0.33 \times 0.4}{0.233} = 0.5714$$

Consequently, the best estimate of the true beta, conditional on observing a beta estimate of 0.9 is:

$$E[\beta | \hat{\beta} = 0.9] = 0.8 \times 0.1429 + 0.9 \times 0.2857 + 1.0 \times 0.5714 = 0.94$$

Bayes' Rule can be applied to all beta estimates, giving the following forecasts conditional on beta *estimates*:

Beta estimate	Expected true beta
0.70	0.80
0.80	0.87
0.90	0.94
1.00	1.00
1.10	1.06
1.20	1.13
1.30	1.20

In summary, beta estimates must be adjusted towards one to eliminate statistical bias in accordance with Bayes' Rule. This is a well-known statistical correction for estimation error.

Note that this adjustment is required even though estimation error is symmetric. In the example above, it is more likely that the true beta is 1.0 (with estimation error of -0.1) than 0.8 (with estimation error of 0.1). This is because more stocks have a true beta of 1.0 than 0.8, and estimation error is symmetric.

The Vasicek adjustment is simply an application of Bayes' Rule to the case of continuous probability distributions, rather than the simple discrete distributions used in this illustrative example.

9. Terms of reference and qualifications

This report was prepared by Professor Stephen Gray, Dr Jason Hall, Professor Robert Brooks and Dr Neil Diamond. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond have made all they enquiries that they believe are desirable and appropriate and that no matters of significance that they regard as relevant have, to their knowledge, been withheld.

Professor Gray, Dr Hall, Professor Brooks and Dr Diamond have been provided with a copy of the Federal Court of Australia's "Guidelines for Expert Witnesses in Proceeding in the Federal Court of Australia." The Report has been prepared in accordance with those Guidelines, which appear in the terms of reference.